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Jessica P. Conry Young
Arkansas Tech University, jyoung35@atu.edu

Samantha Dix
Arkansas Tech University, Sdix@atu.edu

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Cover Page Footnote
We would like to thank Arkansas Tech University for the funding to complete this project.

Generation of Generalized-Gauss Laser Beams via a Spatial Light Modulator

S. Dix and J.P.C. Young*

Department of Physical Science, Arkansas Tech University, Russellville, AR 72802

*Correspondence: jyoung35@atu.edu

Running title: Generalized-Gauss Laser Beams via a Spatial Light Modulator

Abstract

Generalized-Gauss laser beams can be described as a continuous transition between the well-known Hermite-Gauss (HG) and Laguerre-Gauss (LG) laser beams. A spatial light modulator (SLM) was made by removing the liquid crystal display (LCD) from an overhead projector. The homemade SLM, encoded with a computer-generated hologram, was then used to convert a fundamental Gaussian beam from a small frame Helium-Neon (HeNe) laser into several different orders of Generalized-Gauss (GG) beams. The experimentally generated GG beam profiles matched the theoretically expected profiles.

Introduction

Controlling the intensity profiles and polarization of laser beams are of interest in applications such as material processing, optical communication, optical computing, and optical tweezing (Dickey *et al*. 2005). Laser beams are light waves and therefore can be described in terms of solutions to the Paraxial Wave Equation (PWE). In Cartesian coordinates, the solution to the wave equation leads to the HG modes (Siegman 1986):

$$\psi_{nm}^{HG}(x,y,z) = A_{nm}^{HG}\frac{w_o}{w(z)}e^{-(x^2+y^2)\left(\frac{1}{w^2(z)}+\frac{ik}{2R(z)}\right)}\,e^{-ikz} \times H_n\left(\frac{\sqrt{2}x}{w(z)}\right)H_m\left(\frac{\sqrt{2}y}{w(z)}\right)e^{-i(n+m+1)\theta(z)}, \quad (1)$$

where A_{nm} is a normalization constant, H_m is a Hermite polynomial of order m, $w(z)$ is the radius of the beam, w_o is the radius of the beam at its focus, and $k = 2\pi/\lambda$ is the wave number for wavelength λ. $R(z)$ is the radius of curvature of the wavefront and given by

$$R(z) = z + \left(\frac{z_R}{z}\right)^2, \quad (2)$$

where

$$z_R = \frac{\pi w_o}{\lambda}, \quad (3)$$

is the Rayleigh range, and

$$\theta(z) = \arctan\left(\frac{z}{z_R}\right), \quad (4)$$

is the Gouy phase. The indices n and m determine the number of lobes in the x and y directions, respectively. Figure 1a shows the beam profile of an HG_{20} beam, note n+1 nodes in the x-direction (horizontal) and m+1 nodes in the y-direction (vertical). In cylindrical coordinates, the solution to the wave equation leads to LG laser modes (Siegman 1986),

$$\psi_{nm}^{LG}(r,\phi,z) = A_{nm}^{LG}\frac{w_o}{w(z)}e^{-r^2\left(\frac{1}{w^2(z)}+\frac{ik}{2R(z)}\right)}\,e^{-ikz} \times L_{\min(n,m)}^{|n-m|}\left(\frac{2r^2}{w^2(z)}\right)(-1)^{\min(n.m)}\,e^{-i(n-m)\phi} \times \left(\frac{r\sqrt{2}}{w}\right)^{|n-m|}e^{-i(n+m+1)\theta(z)}, \quad (5)$$

where L_μ^ν is the generalized Laguerre polynomial. The smaller of n and m plus one ($\min(n,m)+1$) gives the number of rings; this is demonstrated in Figure 1c which shows the intensity distribution of an LG_{20} beam.

HG and LG modes are complete and orthogonal solutions to the wave equation; therefore, they can be written in terms of each other (Beijersbergen *et al.* 1993),

$$\psi_{nm}^{LG}(x,y,z) = \sum_{k=0}^{N=n+m} i^k b_k^{nm}\psi_{N-k,k}^{HG}\,, \quad (6)$$

where the coefficients

$$b_k^{nm} = \sqrt{\left(\frac{(n+m-k)!\,k!}{2^{n+m}\,n!\,m!}\right)}\frac{1}{k!}\frac{d^k}{dt^k}\left[(1-t)^n(1+t)^m\right]_{t=0}. \quad (7)$$

Note that the i^k term in the sum of Eq. (6) provides a $\pi/2$ phase difference between each term in the series.

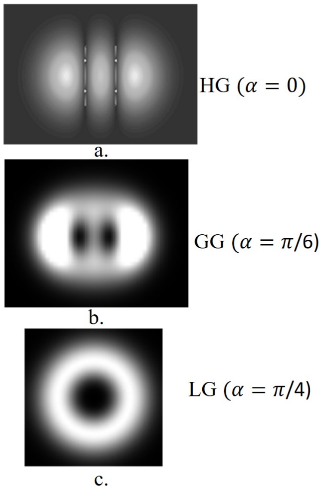

Figure 1: Intensity profiles for a a) Hermite-Gauss (HG), b) Generalized-Gauss (GG), and c) Laguerre-Gauss (LG) beam of order (n,m)=(2,0). A GG beam becomes an HG beam for $\alpha = 0$, and it becomes an LG beam for $\alpha = \pi/4$.

Additionally, an HG beam rotated $\pi/4$ (45°) to the axis can be written as the same sum as the LG beams but the successive terms are in phase (no i^k term):

$$H_{nm}\left(\frac{x+y}{\sqrt{2}}, \frac{x-y}{\sqrt{2}}, z\right) = \sum_{k=0}^{N=n+m} b_k^{nm} \psi_{N-k,k}^{HG}. \tag{8}$$

Figure 2 pictorially demonstrates Eqs. (6) and (8) for the mode composition of HG_{10} rotated 45° (Fig 2a) and LG_{10} (Fig 2b). Note that LG_{10} and HG_{10} rotated 45° are composed of the same HG modes, but LG_{10} has a phase difference of $\pi/2$ between the terms.

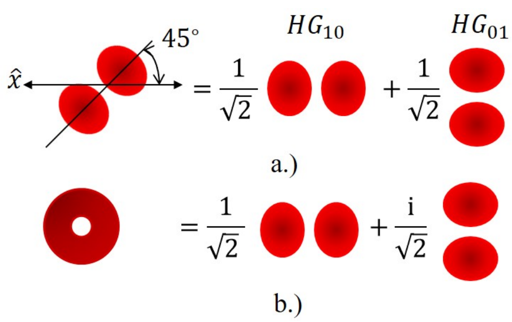

Figure 2: Mode composition of a) HG_{10} rotated 45° and b) LG_{10}.

Laguerre-Gauss beams can be generated from HG beams by exploiting this phase difference. An astigmatic mode converter (AMC) can introduce the i^k phase difference between each term by introducing an astigmatism that will force the Gouy phase to differ by $\pi/2$ in each sequential term (Beijersbergen *et al.* 1993). Figure 3 shows an AMC that consists of two cylindrical lenses of focal length f, separated by a distance $d = \sqrt{2}\, f$ and placed so that the beam waist location is at the midpoint between the two cylindrical lenses. An HG beam, incident on an AMC, will be converted to an LG beam of the same order provided the HG mode is rotated 45° with respect to the axis of the cylindrical lens.

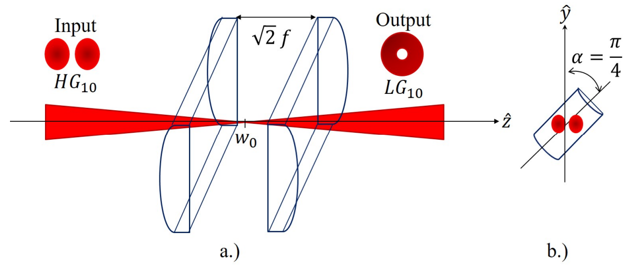

Figure 3: a) An AMC consisting of two cylindrical lenses of focal length f and separated by a distance $2f$. b) The axis of the input (HG) mode must be 45° with respect to the axis of the cylindrical lens for conversion from HG to an LG mode.

A more general solution to the PWE is developed if the angle between the AMC and the input HG mode is at an arbitrary angle, α. This leads to a third solution to the PWE that produces GG beams (sometimes referred to as Hermite-Laguerre-Gauss beams). This third family of solutions can be written in terms of HG beams and the angle α (Abramochkin and Volostnikov 2004):

$$\begin{aligned}\psi_{nm}^{GG}(x,y,\alpha) = \sum i^k \cos^{n-k}\alpha \sin^{m-k}\alpha \\ \times P_k^{n-k,m-k}(-\cos 2\alpha) \\ \times \psi_{n+m-k,k}^{HG}(x,y,z)\end{aligned} \tag{9}$$

where

$$\begin{aligned}P_k^{ab}(\mu) = \frac{(-1)^k}{2^k k!}(1-\mu)^{-a}(1+\mu)^{-b} \\ \times \frac{d^k}{d\mu^k}[(1-\mu)^{k+a}(1+\mu)^k,\end{aligned} \tag{10}$$

are Jacobi Polynomials. It is important to note here that that $0 \le \alpha \le \pi/4$ and when α equals zero the GG beam becomes an HG beam and when α is $\pi/4$ the beam becomes an LG beam. The GG beams can be thought of as a continuous transition between HG and LG beams.

Figure 1 illustrates this transition for $HG \rightarrow GG \rightarrow LG$ beams of order $n = 2, m = 0$.

The solution to the wave equation in elliptical coordinates leads to Ince-Gauss (IG) beams (Bandres and Gutiérrez-Vega 2004). IG beams are also a continuous transition between HG and LG beams in that when the ellipticity of the beam is zero (the elliptic coordinates become circular coordinates) the beam is an LG mode and as the ellipticity of the beam goes to infinity (the elliptic coordinates become Cartesian coordinates) the beam is an HG mode. There is no reported direct relationship between IG and GG beams.

Astigmatic mode converters produce high mode purity beams but are tedious to align and are sensitive to slight disturbances. Additionally, higher order HG beams are typically created by inserting a fiber inside the laser cavity (Beijersbergen *et al.* 1993). Therefore, the mode order is limited by the diameter of the cavity, and it is not often practical to have access to the inside of a laser cavity. In this work, spatial light modulation was used as an alternative method to generate Generalized-Gauss laser beams. Beam shaping using a spatial light modulator only requires that a fundamental (lowest order) Gaussian beam be produced by the laser cavity to externally generate any order GG beam with arbitrary α.

Beam shaping and experimental setup

A transmission spatial light modulator (SLM) can alter the phase, amplitude, and polarization of a wavefront by allowing light to either pass through each pixel or not depending on the input light's polarization (Dickey *et al.* 2005). In this work, a transmission SLM was made by removing the liquid crystal displays (LCDs) and light bulb from a surplus overhead projector (Epson Powerlite 83C), after overriding the missing hardware warnings, one LCD was reconnected to the projector using a flexible printed circuit (FPC) extension board and cable (Huang *et al.* 2012). Figure 4 is a photograph of the homemade SLM and the FPC extension board used to increase the cable length so the LCD could be used outside the projector casing. The SLM was addressed by the computer by treating the SLM as an additional monitor, replicating the computer screen on the SLM.

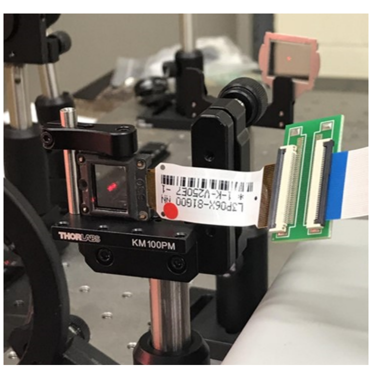

Figure 4: Photograph of the spatial light modulator and the flexible printed circuit extension board and cable.

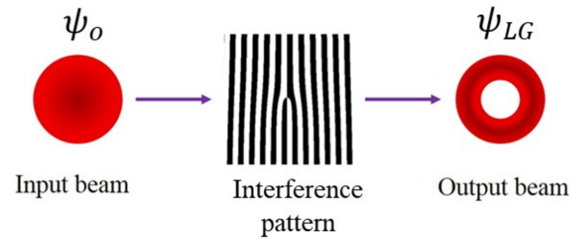

Figure 5: The interference pattern resulting from a fundamental Gaussian (ψ_o) and a Laguerre-Gauss (ψ_{LG}) beam interfering with each other.

Acting as a hologram, the SLM can, in theory, convert the intensity distribution of a beam into a beam with any desired profile provided the intensity distribution of the input beam is known. The hologram encoded on the SLM must be the interference pattern created when the input beam and desired output beam interfere. As an example, consider the fundamental Gaussian input beam and a desired donut shaped LG output beam shown, as shown in Figure 5. The pattern shown at the center of the figure is a result of the fundamental Gaussian beam interfering with the LG beam. Mathematically, the inference pattern is the intensity of the superposition of the two waves,

$$I = |\psi_o + \psi_{LG}|^2, \tag{11}$$

where ψ_o is the wave function of the fundamental Gaussian beam and ψ_{LG} is the wave function of the LG beam. When the interference pattern shown in Figure 5 is encoded on the SLM, and a fundamental Gaussian beam is passed through, the desired LG beam will be produced as an output. The program written to produce the holograms required to generate the GG beams was written in the open source science programing language, GNU Octave and based on code published by Rosales-Guzmán and Forbes (Rosales-Guzmán *et al.* 2017)

The experimental setup used to generate GG beams is shown in Figure 6. A HeNe laser operating in a fundamental Gaussian mode was first sent through a linear polarizer with its transmission axis aligned with the SLM, then the beam was sent through the SLM, a second polarizer, and then to the charged couple device (CCD) camera. The purpose of the second polarizer was to decrease the intensity of the GG beam so the CCD would not be saturated. The SLM was addressed by one laptop computer with a VGA output and the signal from the CCD was read via a USB cable by a second laptop computer. The use of two computers was necessary for this arrangement because it was not possible to have real-time observation of the beam profile acquired by the CCD on the computer screen and have the screen (and SLM) display only the hologram at the same time.

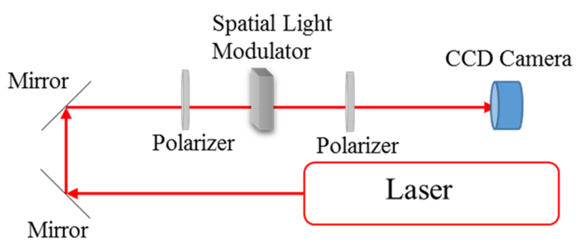

Figure 6: Diagram of the experimental setup consisting of a HeNe, laser, two polarizers, a CCD camera, and two laptop computers.

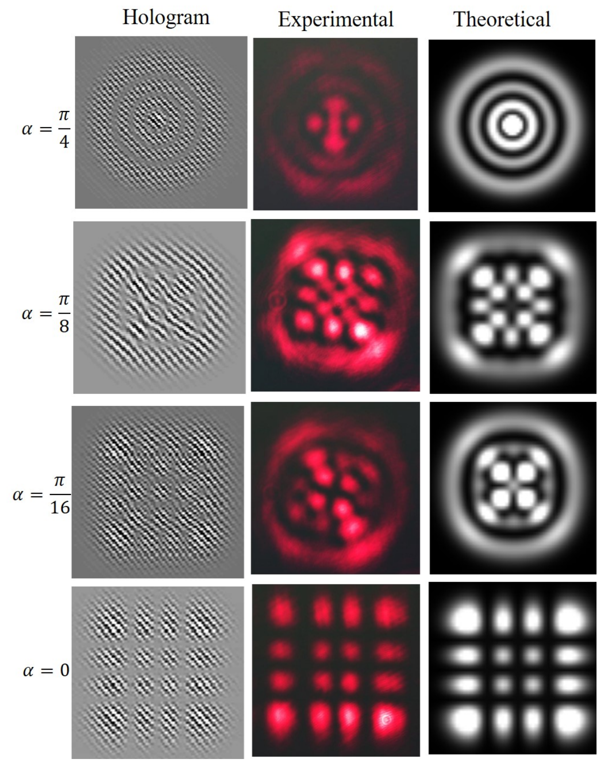

Figure 7: The holograms (first column) used to produce Generalized-Gauss beams of order GG_{33}. The second and third columns are the experimental and theoretically expected beam profiles for the GG_{33} mode, respectively for $\alpha = \pi/4$, $\pi/8, \pi/16$, and 0.

Results

The beam profiles of the GG_{33} beam acquired by CCD are shown in Figure 7. The first column of Figure 7 are the holograms that were encoded on the SLM to generate the corresponding beams. While profiles for one mode order for a few values of α are shown here, we were able to generate any order GG beam, with reasonable quality, up to $(n, m) = (5,5)$ with the setup described in the previous section. The mode order is only limited by the resolution of the SLM. Using a lens to change the beam size will effectively increase the usable number of pixels on the SLM, and higher order modes could be generated. It is important to note that, as expected, as α goes to $\pi/4$ the GG beam transitions to an LG beam with cylindrical symmetry and as α goes to zero the GG beam becomes an HG beam with Cartesian symmetry. The experimental and theoretical beam profiles shown in Figure 7 are in reasonable agreement. Some difference between the experimental and theoretical profiles are observed such as a four lobed pattern at the center of the experimental profile for $\alpha = \pi/4$ compared to the ring in the theoretical profile. The primary reason for this and other observed differences is that the SLM is programmed assuming that that input beam was a pure fundamental Gaussian mode. The laser used does not have the ideal perfectly Gaussian intensity profile. Improving the quality of the input beam or compensating for the slightly non-Gaussian profile when programming the SLM would improve the correlation between the theoretical and experimental beam profiles.

Conclusion

We have shown that Generalized-Gauss laser beams of several orders can be generated using a computer-generated holograms encoded on a homemade SLM. The experimentally generated beams match well with the theoretically expected profiles.

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